

Thermal efficiency of MHD micropolar penta-hybrid nanofluid squeezing flow under nonlinear thermal radiation and activation energy effects

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ABSTRACT

The dynamics of magnetohydrodynamic (MHD) squeezing flow and heat transfer have broad industrial and engineering applications, including micro-electromechanical systems, lubrication technology, squeeze-film dampers, extrusion, rolling, coating processes, and biomedical engineering, where blood flow in narrow arteries is of interest. Motivated by these applications, this study examines the MHD micropolar squeezing flow and thermal behavior of a nonlinear radiative penta-hybrid nanofluid between two parallel disks under the influence of activation energy. To enhance thermal efficiency, the nanofluid is formulated by dispersing copper (Cu), silver (Ag), gold (Au), magnesium oxide (MgO), and titanium dioxide (TiO₂) nanoparticles in a water base fluid. The penta-hybrid nanofluid comprises Ag, Au, MgO, Cu, and TiO₂ nanoparticles, with equal volume fraction distribution. The governing partial differential equations are reduced to nonlinear ordinary differential equations via similarity transformations and solved using the Spectral Weighted Residual Technique (SWRT). Graphical and tabular results reveal the effects of key parameters on velocity, microrotation, temperature, and concentration profiles, providing insights for optimizing advanced thermal and industrial processes. Increasing the magnetic field decreases fluid velocity but improves heat energy transfer, micropolar effects suppress flow intensity, improving stability and momentum transfer in industrial applications, and higher chemical reaction rates reduce solute concentration, while increased activation energy weakens the concentration gradient.

1. Introduction

Nanofluids, which are engineered by suspending nanoparticles in a base solvent, have garnered noteworthy attention for their higher thermophysical properties and improved heat transfer performance in modern thermal engineering systems. Following the initial classical model of Choi and Eastman [1], the introduction of nanoparticles, including carbides, metal oxides, metals, and carbon-based materials, has been established as a proficient strategy for enhancing the conductivity and thermal effectiveness of base fluids [2,3]. Subsequent investigations by Tiwari and Das [4], Khan [5] and Khan et al. [6] confirmed heat transfer augmentation in nanofluids under diverse flow regimes. To further maximize the thermal transport properties, scholars introduced hybridized nanofluids as a higher alternative. Suresh et al. [7] experimentally established that Al₂O₃-Cu/water hybridized nanofluids perform better thermally than unitary-nanofluids. Expanding this concept, Alao et al. [8] numerically computed the mixed

convection and heat radiation effect of electro-magnetized Tiwari-Das hybrid Al₂O₃ and TiO₂ nanofluid model in a permeable microchannel. It was stated that the heat conduction and propagation of the fluid are augmented by about 2.37%, demonstrating better thermal transport. Hansda [9] investigated the influence of inclined magnetic field and thermal radiation on the hybridized nanofluid mixed convective thermosolutal in an active wavy partially trapezoidal plate. With different dynamics terms, the conducting fluid temperature distribution is found to increase progressively as the volume fraction is raised. The idea of hybrid nanofluid has recently been extended to tri, tetra, and penta-hybrid with the aim of enhancing thermal conductivity properties of the base fluid for an improved industrial productivity.

Currently, the development of hybrid and multi-hybrid nanofluids having two or more diverse nanoparticles has advanced the field through synergistic nanoparticle interactions, leading to more momentous thermal performance [10–12]. Penta-hybrid nanofluids consider

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five different kinds of nanoparticles, representing the next evolutionary frontier with a multifaceted enhancement in thermal, electromagnetic, and rheological properties, Ganji et al. [13]. In fluid mechanics, the choice and properties of the working fluids are vital in determining the quality of the product. Hence, nanofluids' thermal performance can be strengthened by the choice of the working fluid's rheological characteristics, which are either Newtonian or non-Newtonian fluids. Traditional Newtonian models do not capture or describe the microstructural complexity behaviors of advanced industrial fluids, unlike a non-Newtonian micropolar fluid [14,15]. The micropolar fluid theory, as presented by Eringen [16], gives a better insight into non-Newtonian fluids, which possess microrotation and microstructure effects. As such, Hayat et al. [17] extended the theory of micropolar fluid to nanofluids and showed the important impact of the microrotation term on the velocity of the fluid and shear stress distribution. When hybrid nanofluids are combined with micropolar effects, the modeling captures asymmetric shear flow and rotational dynamics, which are applicable in lubrication and microscale cooling systems. Usafzai et al. [18] presented multiple exact solutions to nanofluid micropolar flow past a slip flat surface velocity with heat propagation. Gumber et al. [19] discussed the thermal transport in hybridized micropolar nanofluid flow through a porous plate with thermal radiation. The effect of different nanoparticle shape factors on the flow characteristics is examined, and it was noticed that the shape factors prompted the flow and temperature distribution. Mishra et al. [20] analyzed the shape factor tri-hybrid Boger-micropolar magnetized nanomaterial flow in concentric disks. It was mentioned that the shape factors of the tri-nanoparticles augmented the thermal transport field significantly by minimizing the molecular bonding. Further studies on the flow of nanofluid and hybrid nanofluid can be seen in the literature. Fatunmbi et al. [21] investigated the effects of viscosity and micropolar effects on the flow and heat transfer of MHD nanofluid systems containing microorganisms. Obalalu et al. [22] studied the effects of hybrid nanofluid on the enhancement of thermal transport and efficiency in the application of solar-powered energy ships. Hussain et al. [23] studied the flow of MHD ternary hybrid nanofluid over a stretching sheet with convective boundary conditions. The results showed the enhancement of thermal efficiency due to the interaction of the nanoparticles. Khan et al. [24] studied the effects of multiple slip conditions in the flow of ternary hybrid nanofluid containing water, CuO, MgO, and TiO₂. The results showed the significant effects of slip conditions on the flow and thermal efficiency in the presence of exponential heat sources. Salawu et al. [25] used Buongiorno's model in the flow of magneto bio convection in the flow of Casson nanofluid over a rotating disk. Garmar et al. [26] reported thermal radiation significantly influence flow behavior and heat transfer in MHD micropolar nanofluid stagnation-point flow over a stretching sheet. However, in practical systems such as bioengineering, lubrication, polymer extrusion, and cooling technologies, the behavior of hydromagnetic fluid confined in two parallel plates is of critical importance.

A parallel disk squeezing flow is a conventional model that defines the flow prompted by two approaching surfaces or moving apart from one another. It finds relevance in polymer compression moldings, micro-electromechanical systems, and bearings, Fatunmbi et al. [27]. When such a flow exists in electrically conducting fluids under the magnetic field influence, the resulting magnetohydrodynamic (MHD) interaction is used to modulate heat transfer rates, suppress turbulence, and control fluid motion, Chamkha and Ismael [28]. The analysis of hydromagnetic fluid allows for the control of the flow field in electrically conducting fluids. Thus, Lahmar et al. [29] studied unsteady squeezing heat transfer of nanofluid flow with variable thermal conductivity and an inclined magnetic field. They observed that the presence of a squeezing surface and electromagnetic-induced flow reduces the velocity but enhances nanofluid heat transfer. Ramzan et al. [30] performed an irreversibility analysis of electromagnetohydrodynamic hybrid nanofluid squeezing flow with variable properties, showing that

transverse electric and magnetic fields play a key role in controlling the flow and improving nanoparticle heat transfer. Additionally, the effects of activation energy and thermal radiation become significant in highly reactive or thermally intensive flow systems [31,32]. Non-linear thermal radiation is particularly relevant in processes such as solar energy harvesting, thermal insulation, combustion, and chemical reactions governed by Arrhenius kinetics [3,33]. It substantially influences chemical reactivity, mass diffusion rates, and entropy generation, making it essential for reactive transport and catalytic modeling. Recent studies highlight its impact in complex fluid flows. Fatunmbi and Adeniyi [34] examined steady magneto-micropolar fluid flow over a stretchable sheet, showing that nonlinear thermal radiation strongly influences entropy generation and temperature distribution. Ullah et al. [35] studied nonlinear Williamson nanofluid flow over a solar plate with periodic boundary conditions, demonstrating that turbulent waves and radiative heat transfer significantly affect the flow dynamics for solar thermal energy applications. Khan [36] analyzed modified Eyring-Powell nanofluid flow with gyrotactic microorganisms, revealing that nonlinear radiation and bioconvection jointly alter temperature profiles and microbial dynamics. These studies collectively show that thermal radiation and activation energy strongly impact thermal boundary layer thickness, mass diffusion characteristics, and overall heat transfer performance in nanofluid systems.

This study focuses on the behavior of a magnetized micropolar squeezing flow and its thermal efficiency when using a penta-hybrid nanofluid in parallel disks. While previous research has explored hybrid and non-hybrid nanofluids under various conditions, the combination of MHD effects, nonlinear thermal radiation, and activation energy in a penta-hybrid nanofluid system is still under-explored. This research aims to fill that gap, with applications in areas like magnetic thermal processing, microreactors, and advanced cooling systems. By modeling this system, we aim to better understand the interactions between magnetic fields, nanoparticles, and activation energy, which will help optimize thermal performance and improve energy storage for nanofluid-based micro devices. The Research Questions: How do MHD effects influence the velocity, microrotation, and thermal behavior of a penta-hybrid nanofluid in a squeezing flow between parallel disks? What is the impact of activation energy and nonlinear thermal radiation on the thermal performance and heat transfer characteristics of the nanofluid system? How do the interactions between the different nanoparticles in the penta-hybrid nanofluid enhance the overall heat transfer and fluid dynamics compared to traditional single or hybrid nanofluids? What role does the micropolar fluid behavior play in modifying the velocity profiles and flow stability in the presence of external magnetic fields and thermal gradients?

2. Problem formulation and modeling

The problem examined in this study consists of the flow and heat transfer of a micropolar penta-hybrid nanofluid under the following assumptions that: the flow is steady, incompressible, and occurs between two porous disks. The disks are positioned at $z = \pm\beta$ and subjected to a uniform injection velocity. A uniform magnetic field of strength B_0 is applied normal to the disk surfaces in the $(z - r)$ plane, as illustrated in Fig. 1. The induced magnetic field is considered negligible due to the small magnetic Reynolds number, and polarization-induced electric fields are not accounted for in the current analysis. The model assumes a single-phase, homogeneous penta-hybrid nanofluid with constant effective thermophysical properties and uniformly dispersed nanoparticles. The effects of non-spherical particle geometries and particle aggregation are neglected in this study. It is assumed that the boundary conditions are symmetric, making them appropriate for modeling squeezing flow between parallel disks. At the disk surfaces, the temperatures and concentrations are fixed as T_1, N_1 (top) and T_2, N_2 (bottom). A nonlinear radiative heat flux and internal heat generation are incorporated in the energy field, while

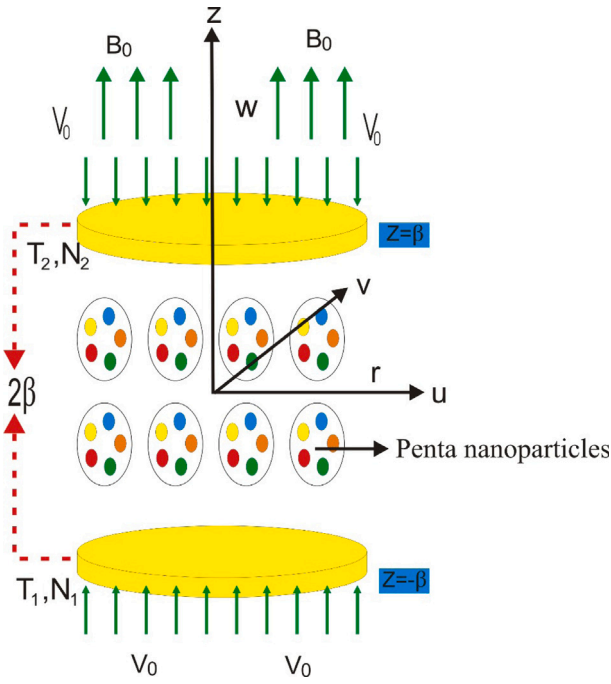


Fig. 1. Flow Configuration.

chemical reaction effects coupled with activation energy are considered in the concentration equation. The penta-hybrid nanofluid consists of five nanoparticles (Ag, Au, MgO, Cu and TiO₂) with respective volume fractions $\phi_1, \phi_2, \phi_3, \phi_4$, and ϕ_5 . The total nanoparticle volume fraction is defined as $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5$. In the present study, equal volume fractions are assumed for simplicity and uniform dispersion, such that $\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi_5 = \frac{\phi}{5}$. The thermophysical properties of the considered nanoparticles are provided in Tables 2 and 3.

2.1. Governing equations

Incorporating the above-highlighted assumptions, the relevant governing equations for the momentum, microrotation, energy, and concentration equations are expressed as follows [37,38].

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho_{phf} \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = (\mu_{phf} + \epsilon) \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{(\mu_{phf} + \epsilon)}{Kp} u - \sigma B_0^2 u - \epsilon \frac{\partial \Omega}{\partial z} - \frac{\partial P}{\partial r} \quad (2)$$

$$\rho_{phf} \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = (\mu_{phf} + \epsilon) \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) + \epsilon \left(\frac{\partial \Omega}{\partial r} + \frac{\Omega}{r} \right) - \frac{\partial P}{\partial z} \quad (3)$$

$$u \frac{\partial \Omega}{\partial r} + w \frac{\partial \Omega}{\partial z} = \frac{\Gamma}{j \rho_{phf}} \left(\frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r} \frac{\partial \Omega}{\partial r} - \frac{\Omega}{r^2} + \frac{\partial^2 \Omega}{\partial z^2} \right) + \frac{\epsilon}{j \rho_{phf}} \left[\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} \right) - 2\Omega \right] \quad (4)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{k_{phf}}{(\rho C_p)_{phf}} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{16\sigma^*}{3k^*(\rho c p)_{phf}} \times \frac{\partial}{\partial z} \left(T^3 \frac{\partial T}{\partial z} \right) + \frac{Q_1}{(\rho c p)_{phf}} (T - T_\infty) \quad (5)$$

$$u \frac{\partial N}{\partial r} + w \frac{\partial N}{\partial z} = Db \left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} + \frac{\partial^2 N}{\partial z^2} \right) - Kr (N - N_1) \times \left(\frac{T}{T_1} \right)^n \exp \left(\frac{-aE}{\zeta T_1} \right) \quad (6)$$

The governing equations (1)–(6) are subject to the boundary conditions:

$$\begin{aligned} u(r, z) = 0, v(r, z) = 0, w(r, z) = V_0, \Omega(r, z) = 0, T = T_1, N = N_1 \text{ at } z = -\beta \\ u(r, z) = 0, v(r, z) = 0, w(r, z) = -V_0, \Omega(r, z) = 0, T = T_2, N = N_2 \text{ at } z = +\beta \end{aligned} \quad (7)$$

All symbols, parameters, and coefficients used in the governing equations are defined with their physical meanings in Nomenclature Table 1. The boundary conditions in Eq. (7) correspond to two parallel plates located at $z = -\beta$ and $z = +\beta$, highlighting the inherent symmetry of the configuration, which is suitable for modeling squeezing flow problems. The plates move toward each other, generating a characteristic squeezing motion of the confined micropolar penta-hybrid nanofluid. Consequently, the flow exhibits a symmetric pattern dictated by the wall motion and reinforced by the imposed temperature difference across the plates.

2.2. Quantities for similarity transformation

The governing equations in partial derivatives are transformed into ordinary derivative by introducing the following similarity transformation quantities as employed by [37–40].

$$\begin{aligned} \psi = \frac{V_0 r^2}{2} f(\eta), \eta = \frac{z}{\beta}, u = \frac{V_0 r}{2\beta} f'(\eta), w = -V_0 f(\eta), \Omega = \frac{V_0 r}{2\beta^2} g(\eta), \\ \theta(\eta) = \frac{T - T_1}{T_2 - T_1}, \phi(\eta) = \frac{N - N_1}{N_2 - N_1} \end{aligned} \quad (8)$$

Upon substituting quantities (8) into the governing Eqs. (1)–(6) with the boundary conditions (7), the underlisted ODEs are obtained:

$$\begin{aligned} (H_1 + K) f^{(4)}(\eta) + H_2 Re f(\eta) f^{(3)}(\eta) - Re M f''(\eta) - K g''(\eta) \\ - \beta_0 Re (H_1 + K) f''(\eta) = 0 \end{aligned} \quad (9)$$

$$G g''(\eta) - Re H_2 B \left(\frac{1}{2} f'(\eta) g(\eta) - f(\eta) g'(\eta) \right) + K (f''(\eta) - 2g(\eta)) = 0 \quad (10)$$

$$\begin{aligned} [H_3 + Nr ((\theta r - 1)\theta(\eta) + 1)^3] \theta''(\eta) + 3 ((\theta r - 1)\theta)^2 (\theta r - 1) \theta'^2 \\ + H_4 Pr Re f(\eta) \theta'(\eta) + Pr Re Q \theta = 0 \end{aligned} \quad (11)$$

$$\phi'' + Sc Re f \phi' - Cr [(\theta r - 1)\theta + 1]^n \exp \left(\frac{E}{(\theta r - 1)\theta + 1} \right) \phi = 0 \quad (12)$$

where the nanofluid parameters are defined as: $H_1 = \frac{\mu_{phnf}}{\mu_f}$, $H_2 = \frac{\rho_{phnf}}{\rho_f}$, $H_3 = \frac{k_{phnf}}{k_f}$, $H_4 = \frac{(\rho c p)_{phnf}}{(\rho c p)_f}$.

The accompanied conditions at the wall transform to

$$\begin{aligned} f'(\eta) = 0, f(\eta) = -1, g(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0, \text{ at } \eta = -1 \\ f'(\eta) = 0, f(\eta) = 1, g(\eta) = 0, \theta(\eta) = 1, \phi(\eta) = 1, \text{ at } \eta = +1 \end{aligned} \quad (13)$$

The governing equations are transformed into dimensionless form using the standard similarity variables [37–40], and all resulting dimensionless parameters are clearly as given in Eq. (14), while the symbolic description of the parameters are written in Table 1.

$$\left\{ \begin{aligned} K &= \frac{\epsilon}{\mu_f}, Re = \frac{\beta V_0}{\nu_f}, M = \frac{\sigma B_0^2 \beta}{\rho_f V_0}, G = \frac{\Gamma}{\mu_f \beta^2}, B = \frac{j}{\beta^2}, \\ Nr &= \frac{16\sigma^* T_1^3}{k^* k_f}, Pr = \frac{(\mu C_p)_f}{k_f}, \theta r = \frac{T_2}{T_1}, \beta_0 = \frac{\nu_f \beta}{K p V_0} \end{aligned} \right. \quad (14)$$

Quantities of Engineering Interest

The Skin friction coefficient (SFC) parameter, couple stress at lower and top disks, Nusselt number, and Sherwood number are orderly described Eqs. (15)–(20):

$$C_{f1} Re_r = (H_1 + K) f''(-1), \quad (15)$$

$$C_{f2} Re_r = (H_1 + K) f''(1) \quad (16)$$

Table 1
Symbolic description of letters used.

Symbols	Description	Symbols	Description
u, w	Velocity components r, z directions	T	Temperature
ν_f	Nanofluid kinematic viscosity	T_2	Temperature at the upper disk
ρ_f	Nanofluid fluid density	T_1	Temperature at the lower disk
μ_f	Nanofluid viscosity	Pr	Prandtl number
k_f	Nano fluid thermal conductivity	θr	Nonlinear thermal radiation parameter
μ_{phf}	Hybrid nanofluid viscosity	ρ_{hnf}	Hybrid nanofluid density axis
$(\rho C_p)_{phf}$	Hybrid nanofluid heat capacity	σ_f	Nanofluid electrical conductivity
k_{phf}	Hybrid nanofluid thermal conductivity	ϵ	Vortex/micropolar viscosity parameter
σ	Electrical conductivity	T_∞	Free stream temperature
$N_1(N_2)$	Concentration at (upper) disk	Ω	Microrotation component
$(\rho C_p)_f$	Nanofluid heat capacity	Q_1	Coefficient of heat generation
Kr	Chemical rate of reaction	K	Micropolar material term
N_2	Concentration at upper disk	N	Concentration
M	Magnetic field term	Nr	Radiation parameter
B	Micro-inertial density term	E	Activation energy parameter
Pr	Prandtl number	G	Is the spin gradient viscosity parameter
Kp	Permeability of the porous medium	β_0	Porosity parameter
aE	Activation energy	Cr	Chemical reaction parameter

Table 2
Thermophysical properties of nanoparticles and base fluid (Water) used in the penta-hybrid nanofluid [13,37,41–45].

Material	ρ (kg/m ³)	C_p (J/kg K)	k (W/m K)
Ag	10 500	235	429
Au	19 300	129	318
MgO	3580	852	48
Cu	8933	385	401
TiO ₂	4250	686.2	11.4
H ₂ O	997	4180	0.613

$$C_{Gr1} Re_r = -Gg'(-1), \quad (17)$$

$$C_{Gr2} Re_r = -Gg'(1) \quad (18)$$

$$Nu(Re_r)^{-1} = -[1 + Nr((\theta r - 1)\theta(\eta) + 1)^3] \theta(\eta)|_{\eta=\pm 1} \quad (19)$$

$$Sh(Re_r)^{-1} = -\phi'(\pm 1) \quad (20)$$

3. Solution methodology

To obtain the needed solution to the set of the transformed boundary value problem (9)–(13), a Spectral Weighted Residual Technique (SWRT) is employed in this study. The selection of Chebyshev polynomials as the basis functions is based on their superior convergence properties and numerical stability, which make them ideal for approximating the solutions to complex fluid dynamics problems. The number of collocation points (P_η) is determined through a convergence analysis, where increasing P_η improves accuracy, but after a certain threshold ($P_\eta \geq 15$), the solution stabilizes, making it computationally efficient to stop further increases. Using this technique, an approximate solution is proposed for the dimensionless quantities $f(\eta)$, $g(\eta)$, $\theta(\eta)$, and $\phi(\eta)$ as expressed below:

$$\begin{aligned} f(\eta) &\approx f^{P_\eta} = \sum_{i=0}^{P_\eta} a_i T_i(\eta), \quad g(\eta) \approx g^{P_\eta} = \sum_{i=0}^{P_\eta} b_i T_i(\eta), \\ \theta(\eta) &\approx \theta^{P_\eta} = \sum_{i=0}^{P_\eta} c_i T_i(\eta), \quad \text{and} \\ \phi(\eta) &\approx \phi^{N_p} = \sum_{i=0}^{N_p} d_i T_i(\eta). \end{aligned} \quad (21)$$

In Eqs. (21), a_i , b_i , c_i , and d_i are constant coefficient to be determined, and $T_i(\eta)$ denote the Chebyshev basis functions. To evaluate the unknown coefficients, Eq. (21) is substituted into the boundary conditions

given in Eq. (13), which leads to Eq. (22).

$$\begin{aligned} f^{P_\eta}(-1) &= 0, \quad f^{P_\eta}(-1) = -1, \quad g^{P_\eta}(-1) = 0, \quad \theta^{P_\eta}(-1) = 0, \\ \phi^{P_\eta}(-1) &= 0, \\ f^{P_\eta}(1) &= 0, \quad f^{P_\eta}(1) = 1, \quad g^{P_\eta}(1) = 0, \quad \theta^{P_\eta}(1) = 1, \quad \phi^{P_\eta}(1) = 1. \end{aligned} \quad (22)$$

The residual errors, $Res_f(\eta, a_i, b_i)$, $Res_g(\eta, a_i, b_i)$, $Res_\theta(\eta, a_i, b_i, c_i)$, $Res_\phi(\eta, a_i, b_i, c_i, d_i)$, are derived by substituting Eq. (21) into Eqs. (9), (10), (11), and (12), respectively. Following the approach of Adeosun and Gbadeyan [46], the collocation method is employed to minimize these residual errors toward zero, which yields a system of $4P_\eta + 4$ nonlinear algebraic equations evaluated at the Gauss–Lobatto points η_i , defined as

$$\eta_i = -\cos\left(\frac{i\pi}{P_\eta}\right) \text{ for } i = 0, 1, 2, \dots, P_\eta.$$

The algebraic equations are solved using the FindRoot function in Wolfram Mathematica 14.1, yielding the values of the unknowns a_i , b_i , c_i , and d_i .

4. Results analysis and deliberations

Table 4 presents the solution convergence of the key physical quantities $-f''(1)$, $g'(1)$, $\theta'(1)$, and $\phi'(1)$ with respect to the collocation parameter P_η using the Spectral Weighted Residual Technique (SWRT). For this analysis, the parameters are fixed at $K = 1$, $Re = 1$, $M = 0.1$, $\beta_0 = 0.1$, $G = 1$, $B = 0.001$, $Nr = 0.1$, $\theta r = 1.2$, $Pr = 7$, $Sc = 1$, $n = 1$, $E = 0.1$, $Cr = 0.1$, $\phi_{1,2,3,4,5} = 0.1$. From the table, a rise in P_η from 5 to 30 reveals that the computed values of all quantities gradually stabilize. This authenticates that a rise in the collocation points enhances the numerical accuracy, and convergence is achieved beyond $P_\eta = 15$. Hence, with $P_\eta \geq 15$ there is a stable and reliable solutions without unnecessary computational cost. Furthermore, Tables 5 and 6 show the comparison of the results obtained for $-f''(1)$ and $g'(1)$ using the Spectral Weighted Residual Method (SWRM) and the shooting method for different values of K and Re . The tables showed both negative and positive Re to account for opposite flow directions ($Re = -10, -5, 10, 5$). From the tables, the SWRM results show excellent agreement with the shooting method and thus, validates the accuracy, stability, and reliability of the SWRM in solving the present nonlinear boundary-layer problem.

Fig. 2 reveals that increasing the magnitude of the micropolar parameter reduces the velocity profile, which indicates a suppression of flow intensity due to the microrotation of suspended particles. Physically, this trend arises from the energy exchange between translation and rotation in micropolar fluids, which absorb part of the kinetic energy of the main flow, thereby reducing axial momentum. This trend is of practical importance in engineering applications such as lubrication systems, journal bearings, polymer processing, suspensions, and biological flows (e.g., blood), where micropolar effects enhance flow

Table 3
Thermophysical characteristics of the penta-hybrid nanofluid components [35,39].

Properties	Formulas
Dynamic viscosity	$\mu_{phnf} = \mu_f [(1 - \phi_1)^{-2.5} (1 - \phi_2)^{-2.5} (1 - \phi_3)^{-2.5} (1 - \phi_4)^{-2.5} (1 - \phi_5)^{-2.5}]$
Density	$\rho_{phnf} = (1 - \phi_5) [(1 - \phi_4) [(1 - \phi_3) [(1 - \phi_2) ((1 - \phi_1)\rho_f + \phi_1\rho_1) + \phi_2\rho_2] + \phi_3\rho_3] + \phi_4\rho_4] + \phi_5\rho_5$
Heat capacity	$(\rho C_p)_{phnf} = (1 - \phi_5) [(1 - \phi_4) [(1 - \phi_3) [(1 - \phi_2) ((1 - \phi_1)(\rho C_p)_f + \phi_1(\rho C_p)_1) + \phi_2(\rho C_p)_2] + \phi_3(\rho C_p)_3] + \phi_4(\rho C_p)_4] + \phi_5(\rho C_p)_5$
Thermal conductivity	$\kappa_{phnf} = \kappa_{thnf} \cdot \frac{\kappa_{thnf} + 2\kappa_5 - 2\phi_5(\kappa_{thnf} - \kappa_5)}{\kappa_{thnf} + 2\kappa_5 + \phi_5(\kappa_{thnf} - \kappa_5)}$
	$\kappa_{thnf} = \kappa_{mhnf} \cdot \frac{\kappa_{mhnf} + 2\kappa_4 - 2\phi_4(\kappa_{mhnf} - \kappa_4)}{\kappa_{mhnf} + 2\kappa_4 + \phi_4(\kappa_{mhnf} - \kappa_4)}$
	$\kappa_{mhnf} = \kappa_{hnf} \cdot \frac{\kappa_{hnf} + 2\kappa_3 - 2\phi_3(\kappa_{hnf} - \kappa_3)}{\kappa_{hnf} + 2\kappa_3 + \phi_3(\kappa_{hnf} - \kappa_3)}$
	$\kappa_{hnf} = \kappa_{nf} \cdot \frac{\kappa_{nf} + 2\kappa_2 - 2\phi_2(\kappa_{nf} - \kappa_2)}{\kappa_{nf} + 2\kappa_2 + \phi_2(\kappa_{nf} - \kappa_2)}$
	$\kappa_{nf} = \kappa_f \cdot \frac{\kappa_f + 2\kappa_1 - 2\phi_1(\kappa_f - \kappa_1)}{\kappa_f + 2\kappa_1 + \phi_1(\kappa_f - \kappa_1)}$

Table 4
Solution convergence of $-f''(1)$, $g'(1)$, $\theta'(1)$, and $\phi'(1)$ for various P_η at: $K = 1$, $Re = 1$, $M = 0.1$, $\beta_0 = 0.1$, $G = 1$, $B = 0.001$, $Nr = 0.1$, $\theta_r = 1.2$, $Pr = 7$, $Sc = 1$, $n = 1$, $E = 0.1$, $Cr = 0.1$, $\phi_{1,2,3,4} = 0.1$.

P_η	$-f''(1)$	$g'(1)$	$\theta'(1)$	$\phi'(1)$
5	2.04250	0.87307	0.16308	0.39283
10	2.52540	0.87761	0.07762	0.38914
15	2.53168	0.87763	0.07713	0.38912
20	2.53164	0.87763	0.07713	0.38912
25	2.53164	0.87763	0.07713	0.38912
30	2.53164	0.87763	0.07713	0.38912

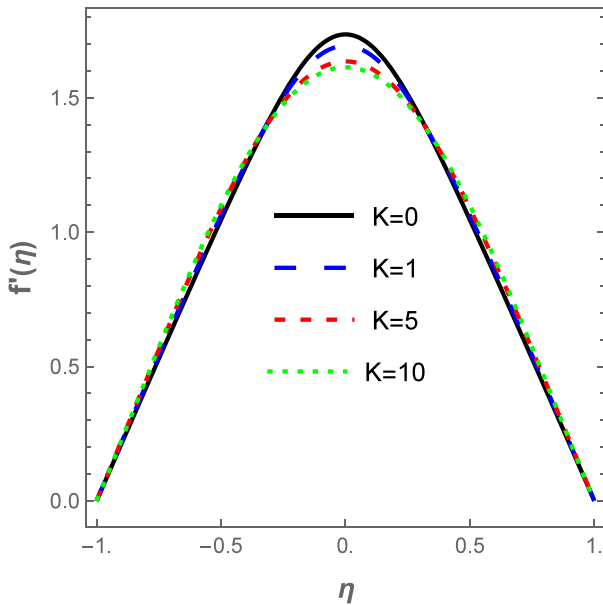


Fig. 2. Velocity curve pattern for varying K .

stability and provide a more realistic description of transport dynamics. Fig. 3 depicts the decline in velocity profiles with increasing magnetic field parameter M . The suppression of fluid motion arises from the Lorentz force, a resistive drag generated by the interaction between the micropolar penta-hybrid nanofluid and the applied magnetic field. This magnetic damping effect is highly relevant in engineering and biomedical systems. For example, in nuclear engineering, magnetic control of liquid-metal coolants enhances reactor safety, while in metallurgical processes, electromagnetic braking regulates molten metal flow during casting to minimize defects and improve product quality.

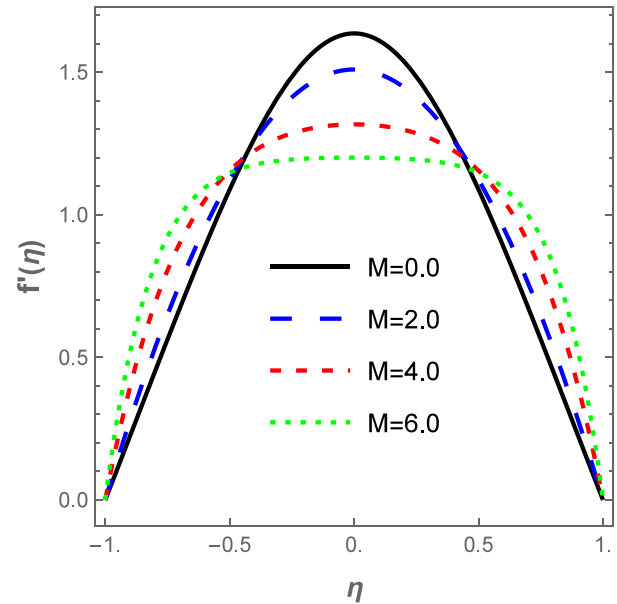


Fig. 3. Velocity profile for rising M .

Fig. 4 reveals that increasing the porosity parameter β_0 suppresses the velocity due to enhanced drag and dissipation of fluid momentum within the porous medium. In practice, porous structures are often exploited to regulate flow dynamics in systems such as filtration, petroleum recovery, geothermal reservoirs, and biomedical transport processes. Fig. 5 shows that the velocity profile broadens as the Reynolds number Re increases. This trend reflects the dominance of inertial forces over viscous resistance at higher Re , resulting in reduced opposition to motion and enhanced momentum transfer within the micropolar penta-hybrid nanofluid. Such behavior is significant in high-speed transport systems, industrial fluid handling, and microchannel processes where efficient momentum transfer is essential.

Fig. 6 shows that the velocity profile magnifies from water to nanofluid, hybrid nanofluid, and penta-hybrid nanofluid, this enhancement results from enhanced thermal conductivity, interfacial slip, and micro-mixing caused by random motion, all of which work together to reduce the viscosity and foster momentum transfer. In the presence of various nanoparticles buoyancy-driven convection becomes strengthened and thus, there is higher velocity dispersion. Thus, hybrid and penta-hybrid nanofluids outperform traditional and single nanofluid, which is applicable for sophisticated heat transfer and energy-saving works. As shown in Fig. 7, the microrotation profile $g(\eta)$ varies with

Table 5

Comparison of the results obtained (in the absence of nanoparticles) for $-f''(1)$ using the Chebyshev spectral collocation method and the shooting method for different values of K and Re at: $M = 0$, $G = 1$, $B = 0.001$.

K	Re				Re			
	-10		-5		10		5	
	SWRM	Shooting	SWRM	Shooting	SWRM	Shooting	SWRM	Shooting
5	3.0200811367	3.0200811151	2.7516865015	2.7516864859	2.2639459258	2.2639459197	2.3826023347	2.3826023276
10	2.5205267364	2.52052667255	2.4020114990	2.4020114461	2.1467485636	2.4020114461	2.2169623806	2.2169623459

Table 6

Comparison of the results obtained (in the absence of nanoparticles) for $g'(1)$ using the Chebyshev spectral collocation method and the shooting method for different values of K and Re at: $M = 0$, $G = 1$, $B = 0.001$.

K	Re				Re			
	-10		-5		10		5	
	SWRM	Shooting	SWRM	Shooting	SWRM	Shooting	SWRM	Shooting
5	3.2836340356	3.2836341274	3.2221829692	3.2221830317	3.0835014894	3.0835015024	3.1223211501	3.1223211732
10	5.0279821125	5.0279823533	4.9489666159	4.9489668200	4.7543250674	4.7543251642	4.8123770823	4.8123772129

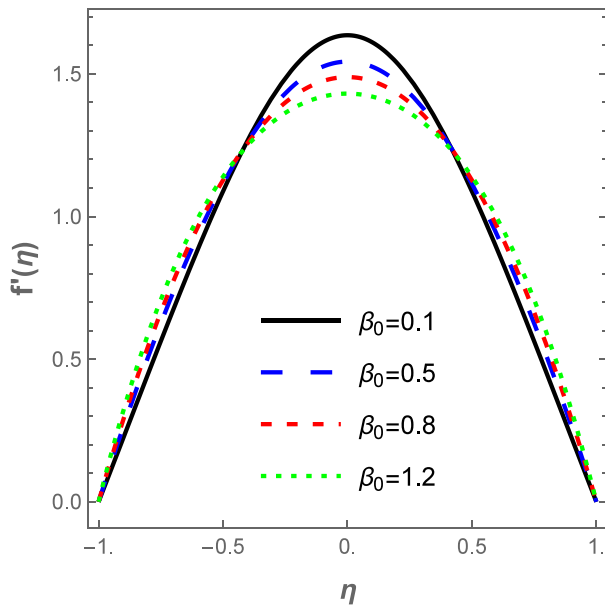


Fig. 4. Velocity profile for variation in β_0 .

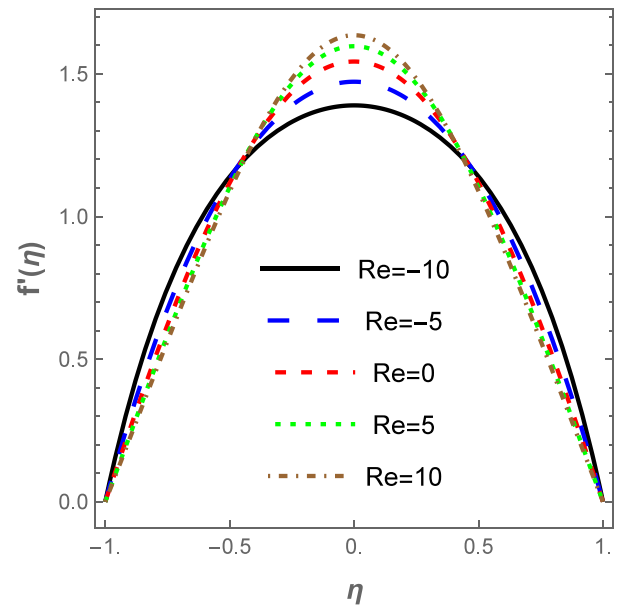


Fig. 5. Velocity curve for growth in Re .

η for different values of K . When $K = 0$, representing the Newtonian fluid case, the microrotation vanishes everywhere. However, as K increases ($K = 1, 5, 10$), larger positive and negative peaks appear in the profile, indicating that stronger micropolar effects intensify the local spin of fluid microelements and enhance the rotational behavior of the flow. These results demonstrate that micropolarity plays a significant role in modifying velocity gradients and rotational motion within the boundary layer.

In Fig. 8, the microrotation profile declines as M escalates ($M = 0, 2, 4, 6$). As found in the figure, the microrotation reaches its maximum amplitude when $M = 0$ whereas an increase in M suppress both the positive and negative peaks. The observed trend is attributed to the Lorentz force which is generated by the magnetic field. This force reduces the fluid flow and consequently at such, there is a damping effect on the rotational motion of the microelements. In this case, a stronger magnetic effects suppresses microrotation intensity within the boundary layer. The Reynolds number's Re impact on the microrotation profile is seen in Fig. 9. As observed, the positive Re values intensify oscillations as inertial forces surpass viscosity but negative Re values repress microrotation, indicating decreased particle rotation and lower microstructural activity. For the micropolar and nanofluid systems,

the observed trend implies that forward flow enhances mixing, heat transmission, and momentum transport whereas reverse flow stabilizes microrotation in the system.

Figs. 10 and 11 respectively showcase the thermal reaction for variation in the magnetic field (M) and heat generation terms (Q). Critical observation reveals that the temperature profile $\theta(\eta)$ increases with higher magnetic parameter M and heat generation parameters. Stronger magnetic fields enhance the Lorentz force, which suppresses fluid motion and raises thermal energy. This leads to higher peak temperatures, indicating that magnetic fields can be effectively used to control heat transfer in micropolar and nanofluid systems. Similarly, increasing Q from 0.1 to 0.8 introduces additional heat into the system, thereby elevating the temperature distribution. The boundary-layer temperature rises steadily and attains a higher peak before declining toward the free-stream value. This behavior results from the internal heat source, which weakens the temperature gradient and releases more thermal energy into the fluid. Such an effect is useful in thermal management applications requiring enhanced thermal fields in nanofluid flow.

Fig. 12 illustrates the variation in the temperature profile as Re escalates when $Q = 0.01$. In the figure, the thermal profile becomes steeper

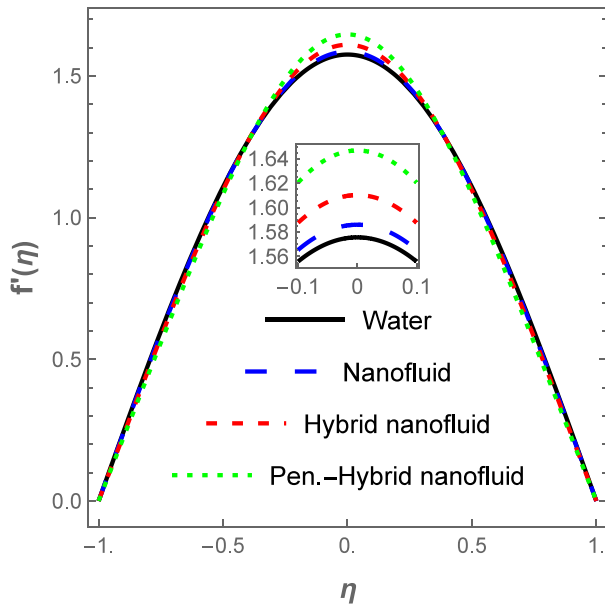


Fig. 6. Velocity for base fluid, nanofluid, hybrid nanofluid, and penta-hybrid nanofluid.

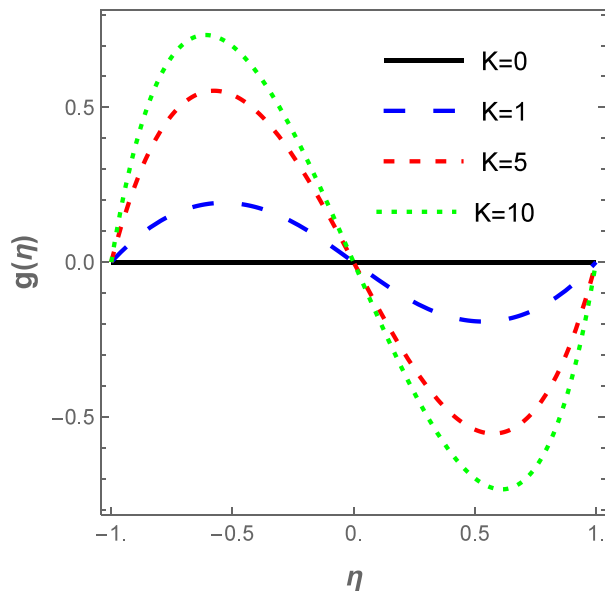


Fig. 7. Microrotation profile for variation in micropolar parameter K .

as Re increased from -3 to 3 . The negative Re suppresses convection and produces lower temperatures while positive Re (assisting flow) increases it and raises the boundary-layer temperature. This demonstrates how inertial forces predominate at high Re fostering more heat transfer in micropolar penta-hybrid nanofluid, which is beneficial for high-speed thermal management applications like microchannel cooling. As shown in Fig. 13, the temperature profile rises monotonically for all working fluids in from the cold side ($\eta < 0$) to the hot side ($\eta > 0$). The water curve assumes the steepest, whereas the curves of nanofluid, hybrid nanofluid, and penta-hybrid nanofluid are increasingly flatter, implying a higher thermal conduction. However, at $\eta = 0$ there is a discernible crossover where all profiles almost coincide. Here, the nanofluid-based systems remain marginally warmer than water for $\eta < 0$, implying faster heat pickup, they cool down somewhat for $\eta > 0$, indicating faster heat disposal. This redistribution improves overall

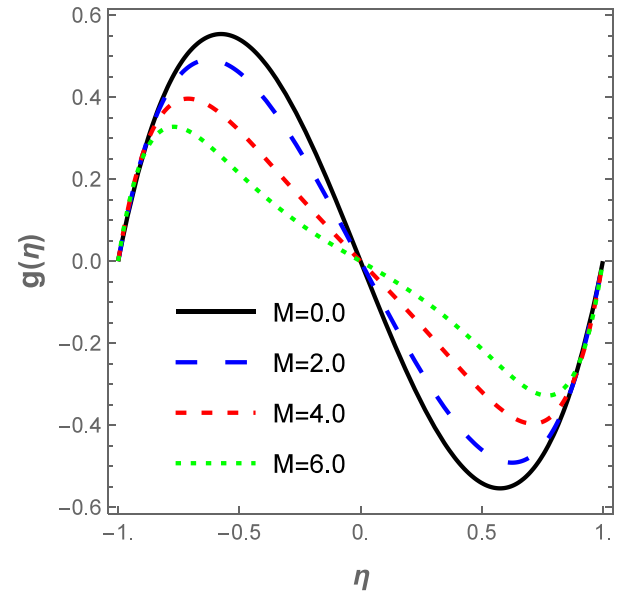


Fig. 8. Microrotation profile for varying M .

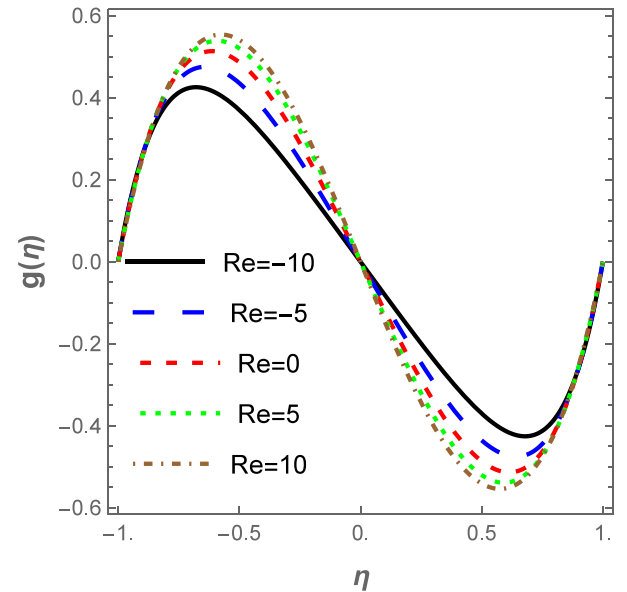


Fig. 9. Microrotation for variation in Re .

heat transmission without altering the boundary temperatures. This trend is applicable in many fields where quick energy absorption on the cold side and efficient dissipation on the hot side improve system efficiency and reduce operating temperatures, for instance, for solar thermal collectors, electronic cooling, and industrial heat exchangers, etc.

A rise in the chemical reaction term Cr from 0.1 to 0.5 suppresses the concentration profile as depicted in Fig. 14. This trend occurs because higher magnitudes of chemical reaction accelerate species consumption; as a result, there is a decline in the solute concentration and a weakening of the diffusion rate across the species boundary layer. The implication of such behavior is common in processes where rapid reactant decrease and improved mass transfer are desired, for instance, in polymerization systems, catalytic reactors, corrosion control, and wastewater treatment operations. Meanwhile, Fig. 15 reveals the impact of the activation-energy term E on the concentration profile. From

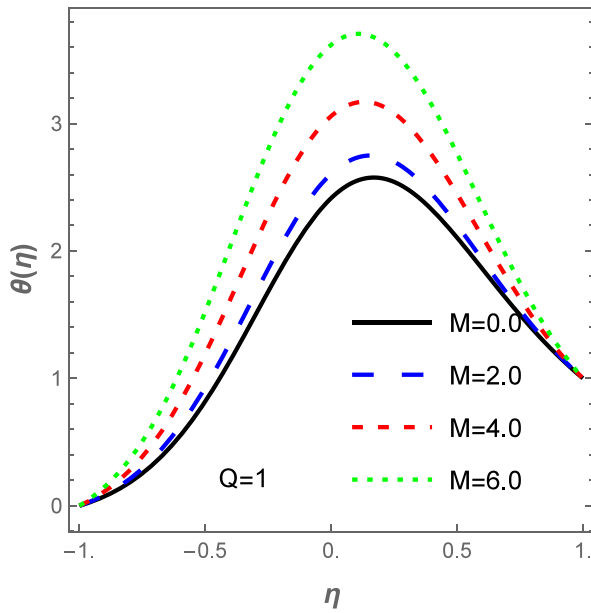
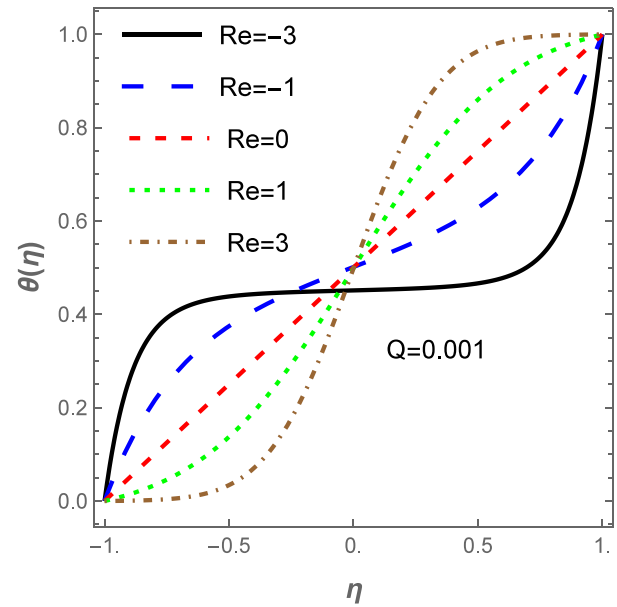
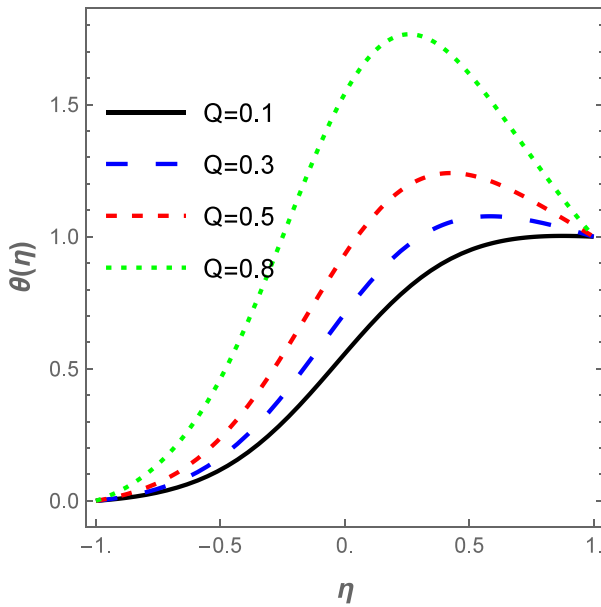
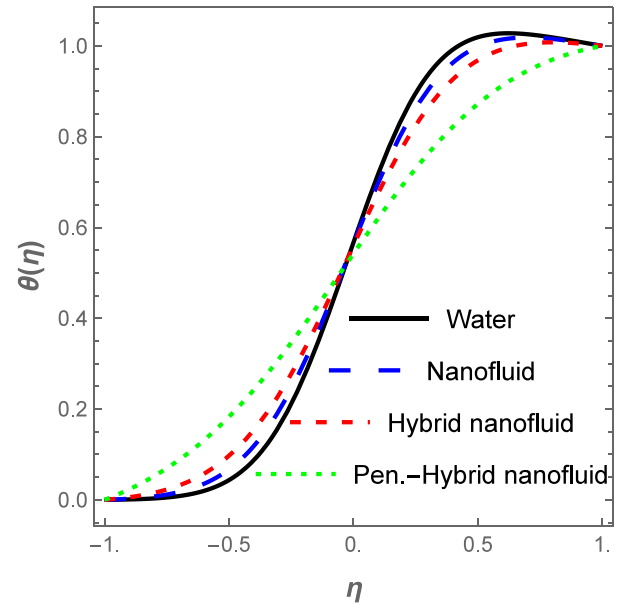
Fig. 10. Thermal profile for varying M .Fig. 12. Thermal field for varying Re .Fig. 11. Thermal profile for growth in Q .

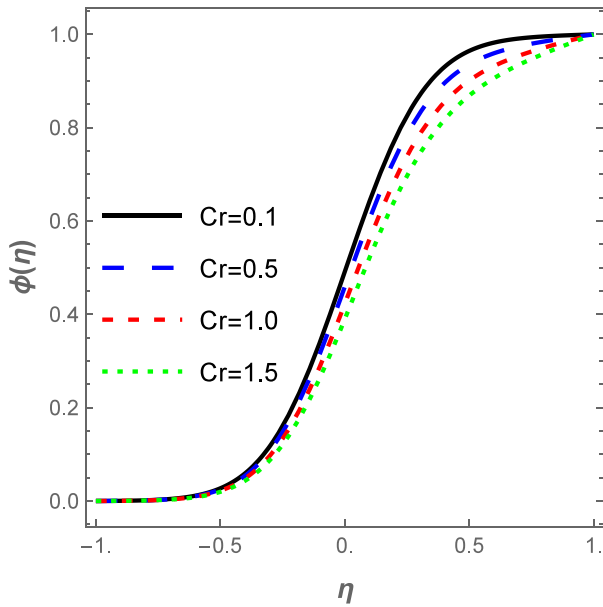
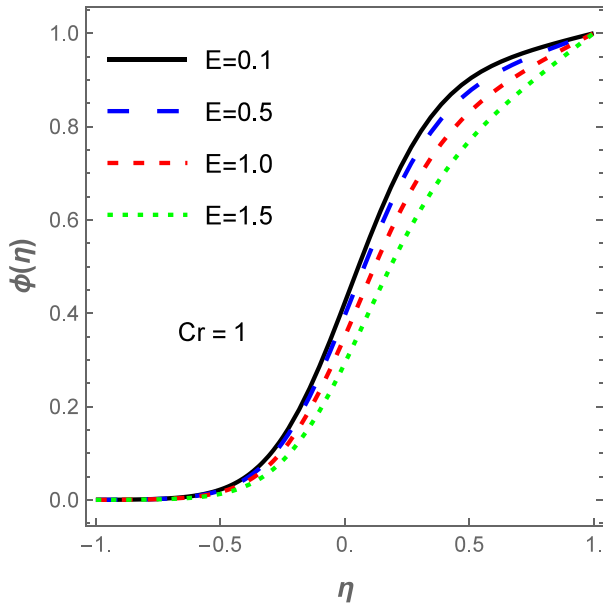
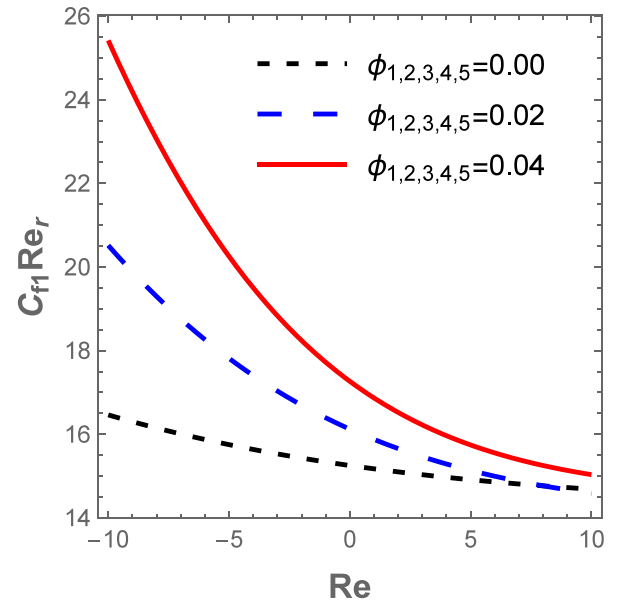
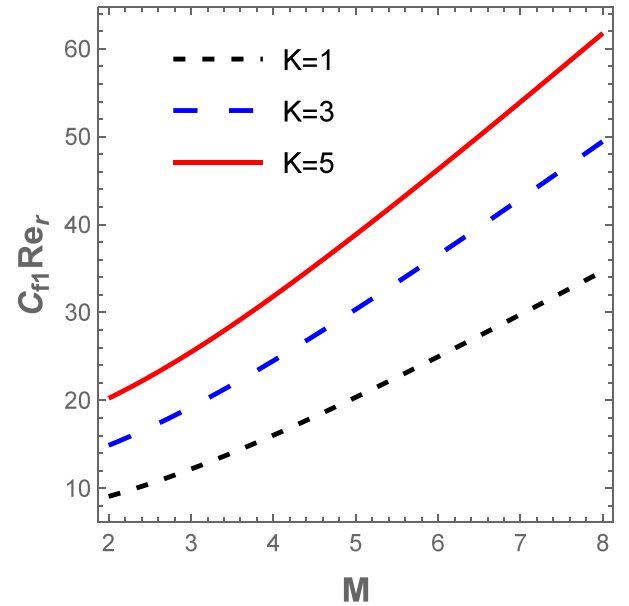
Fig. 13. Thermal profile for base fluid, nanofluid, hybrid nanofluid, and penta-hybrid nanofluid.

this figure, it is observed that for all values of E the concentration profile rises monotonically from the reactant side ($\eta < 0$) to the product side ($\eta > 0$) and approaches unity. It is found that an increase in E leads to a shrinking of the concentration curves, with the solute concentration diminishing at any given point. This trend reflects that, with rising E , there is a rise in the energy barrier for the chemical reaction, and, as such, there is a lower consumption rate and a weakening of the concentration gradient across the layer.

In Fig. 16, the skin friction coefficient (SFC) falls for all values of Re , both negative and positive. This is so because as Re increases, the fluid's inertia dominates viscosity and, as such, the wall experiences less relative friction. Meanwhile, an increase in $\phi_{1,2,3,4,5}$ makes the SFC to rise due to a rise in thermal conductivity which boosts buoyancy and momentum transfer, causing a rise in the velocity gradient and consequently increasing the wall shear. In Fig. 17, an increase in the

magnetic parameter M raises the viscous drag and causes the wall shear stress to increase. A rise in M corresponds to an increase in the strength of the Lorentz force and, as such, there is a drag in the fluid motion near the wall. This additional resistive force slows the fluid, leading to a thickened momentum boundary layer, which directly translates to a higher skin friction coefficient (SFC).

Fig. 18 depicts the combined effects of the magnetic parameter M and the micropolar parameter K on the Nusselt number (Nu). A rise in the magnitude of M suppresses the heat transfer across the surface, whereas an increase in K exhibits the opposite trend. An increase in K corresponds to stronger microrotation, which induces spinning of the micro-elements within the fluid and promotes energy storage. Consequently, growth K strengthens microrotation, raises the thermal gradient, and thus, boost the Nusselt number, indicating improved convective heat transfer between the disks. Fig. 19 reveals that a

Fig. 14. Concentration profile for Cr .Fig. 15. Concentration profile for rising E .Fig. 16. SFC for variation in $\phi_{1,2,3,4,5}$ & Re .Fig. 17. SFC for variation in K & M .

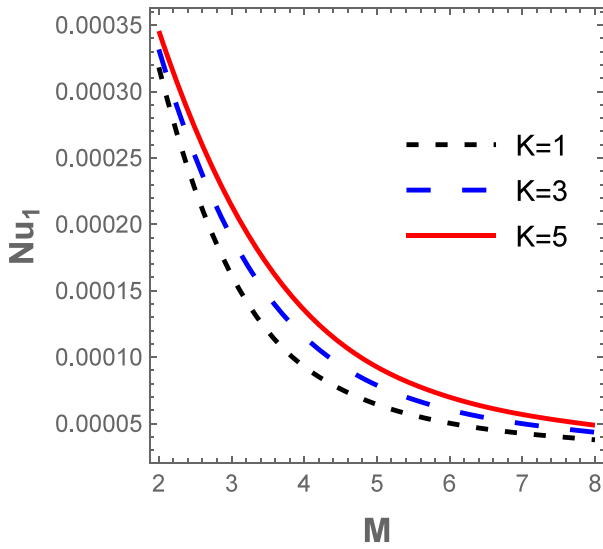
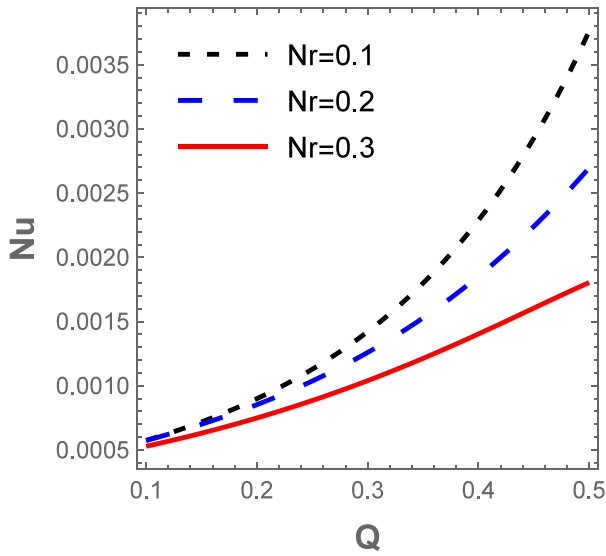
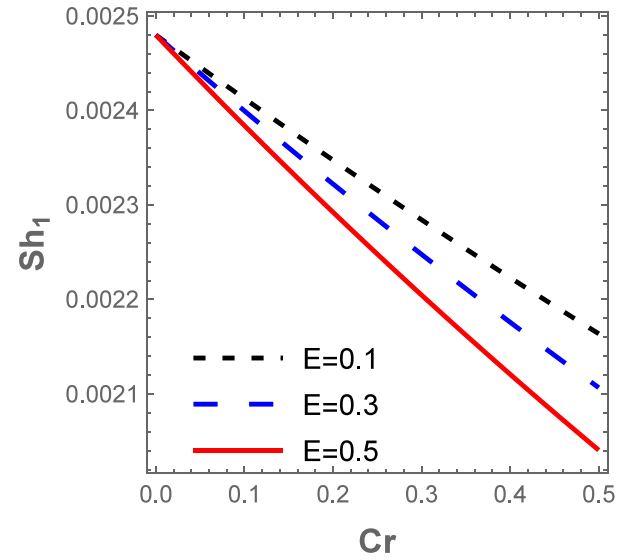
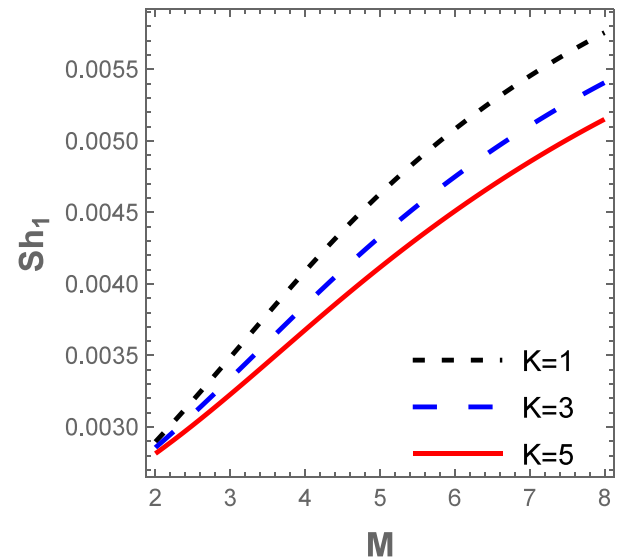
rise in both Nr and Q raises the heat transfer across the surface. An increase in the magnitude of Nr leads to a decline in the heat transfer whereas, a rise in Q enhances internal heat generation, leading to an elevated thermal profile and further intensifying the wall-to-fluid thermal difference and thus, enhance convective heat transfer, resulting in a higher Nusselt number. However, a rise in Nr causes a downward thermal gradient, resulting to a decreased Nusselt number due to a reduction in the convective heat transfer efficiency at the wall (see Fig. 19).

In Fig. 20, the mass transfer phenomenon declines for a rise in E for all values of Cr . Similarly, a rise in Cr causes a fall in the mass transfer as found in this figure. Here, a rise in E raises the energy barrier for the reaction, and thus, slow down the rate of species conversion and lower the concentration gradient across the boundary layer. In a similar way, a rise in Cr strengthens the chemical reaction, which

accelerates species consumption and reduces the solute concentration. The trend in this figure is commonly encountered in real processes like polymerization, catalytic reactors, corrosion control, and wastewater treatment. In these areas, controlling the rates of chemical reaction and activation energy becomes important for optimizing mass transfer efficiency. Fig. 21 displays the impact of parameters K and M on the mass transfer rate, and it was discovered that mass transfer increases with a rise in M values, while it decreases with higher values of K .

5. Conclusion

This study examined the flow dynamics and heat transfer characteristics of a magnetohydrodynamic micropolar penta-hybrid nanofluid squeezed between two parallel disks under the combined effects of chemical reaction, activation energy, and nonlinear radiative flux.

Fig. 18. Nusselt number for rising M & K .Fig. 19. Nusselt number for combined Nr & Q .Fig. 20. Mass transfer for rising Cr & E .Fig. 21. Mass transfer for rising K & M .

Thermal efficiency was significantly enhanced by dispersing five distinct nanoparticles — copper (Cu), silver (Ag), gold (Au), magnesium oxide (MgO), and titanium dioxide (TiO_2) — in a water-based fluid. The governing partial differential equations were transformed into ordinary differential equations and solved using the Spectral Weighted Residual Technique (SWRT). Results, presented in both graphical and tabular forms, show excellent agreement with those obtained via the classical shooting method with a Runge–Kutta scheme, thereby confirming the accuracy, stability, and reliability of the SWRT for analyzing complex nanofluid systems. This research has practical applications in advanced cooling systems, lubrication, and tribology, including the design of high-performance cooling devices for microelectronics, nuclear reactors, and space technologies, as well as squeeze film bearings, automotive lubrication systems, catalytic processes, and heat exchangers. The analysis reveals that:

- The velocity profiles broaden as the magnitude of Reynolds number (Re) escalates due to the dominance of inertial forces over viscous resistance, whereas the porosity (β_0), material (K), and magnetic field (M) parameters suppress the velocity profile.

- There is an enhancement of the velocity profile from water to nanofluid, hybrid nanofluid, and penta-hybrid nanofluid due to the reduction in viscosity caused by enhanced thermal conductivity, interfacial slip, and micro-mixing from random nanoparticle motion.
- The temperature profiles increase with rising magnitudes of M , Re , and Q . Moreover, the temperature distribution propagates across all working fluids, with higher thermal conduction observed in the order: water $\rightarrow$ nanofluid $\rightarrow$ hybrid nanofluid $\rightarrow$ penta-hybrid nanofluid.
- A shrinking concentration profile occurs as the chemical reaction and activation energy terms grow in magnitude due to lower consumption rates and a weakening of the concentration gradient across the layer.
- The skin-friction coefficient (SFC) decreases for all values of Re (both negative and positive), but an increase in $\phi_{1,2,3,4,5}$ boosts thermal conductivity, causing a rise in SFC. Likewise, a rise in M increases the viscous drag.

- The heat transfer rate magnifies with a rise in M , Q , and Nr , whereas an increase in K exhibits the opposite trend.
- The mass transfer declines for higher magnitudes of activation energy (E), chemical reaction rate (Cr), and material parameter (K), whereas an increase in the magnetic parameter (M) enhances the mass transfer.

CRedit authorship contribution statement

E.O. Fatunmbi: Writing – original draft, Formal analysis, Conceptualization. **A.T. Adeosun:** Validation, Methodology. **S.O. Salawu:** Writing – review & editing, Writing – original draft, Formal analysis. **O.A. Olaiju:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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