



Thermofluid-inspired Nonlinear Entropy Model of Financial Markets with Time-varying External Shocks

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study develops a thermofluid-inspired nonlinear entropy model to investigate uncertainty, instability, and risk propagation in financial markets under time-varying external shocks. The model is formulated as a nonlinear reaction–diffusion equation incorporating logistic-type growth, saturation effects, and a space–time-dependent source term representing decaying market disturbances. Through nondimensionalization, key governing parameters associated with diffusion, amplification, and dissipation are identified. Numerical solutions, obtained using an explicit finite difference scheme, reveal that increased diffusion enhances the spatial redistribution of financial entropy, thereby reducing localized instability. Nonlinear saturation limits excessive entropy growth, ensuring bounded system behavior, while external shocks significantly elevate entropy in the short term but exhibit diminishing long-term influence. Furthermore, entropy is shown to attain

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peak values at intermediate market depth and liquidity levels, highlighting a nonlinear balance between information flow and market friction. The results demonstrate that financial entropy dynamics are governed by the interplay of diffusion, nonlinear reactions, and transient shocks. The proposed framework provides a thermodynamically consistent and quantitatively robust approach for modeling market complexity, with potential applications in risk assessment, volatility prediction, and financial system stabilization.

Keywords: *Thermofluid framework; nonlinear entropy modeling; financial markets; time-varying external shocks.*

1. Introduction

Entropy, developed in thermodynamics, is a rigorous measure of irreversibility, disorder, and energy dissipation in physical systems, forming the basis of the Second Law of Thermodynamics (Bejan, 1996). Its development in statistical mechanics and information theory provided a probabilistic link between microscopic randomness and macroscopic variables, making it a powerful tool for analyzing complex systems with uncertainties. This dual thermodynamic–informational perspective has enabled entropy to extend beyond physical systems to diverse fields, including economics and finance. Bejan (2002) established the theoretical foundation for entropy generation minimization and exergy analysis, demonstrating the role of irreversibility in thermodynamic systems. Sciacovelli et al. (2015) further highlighted entropy generation as a key tool for design optimization in thermal systems. In fluid flow applications, Ullah et al. (2021) employed numerical analysis for non-Newtonian fluids, showing the influence of viscosity on entropy generation, while Fatunmbi and Salawu (2020) studied magneto-micropolar fluid flow in porous media, illustrating the effects of magnetic fields and microstructure on entropy production. Fatunmbi and Adeniyi (2020) examined the combined effects of nonlinear thermal radiation and variable properties on entropy generation in stretchable surface flows, demonstrating the significance of nonlinear processes in thermodynamic irreversibility.

Financial markets are complex adaptive systems characterized by nonlinear interactions, random fluctuations, and emergent patterns. Such features have inspired interdisciplinary research, particularly in econophysics, where concepts from statistical physics and information theory are applied to better understand financial dynamics (Mantegna & Stanley, 2000; Rosser, 2021). In this context, entropy naturally quantifies uncertainty, allowing for a deeper characterization of complex distributions. Measures such as Shannon entropy and relative entropy provide a more complete description of uncertainty than traditional variance-based metrics by considering the full probability distribution of returns (Zhou et al., 2013). The principle of maximum entropy has been widely applied for probability estimation and model calibration under incomplete information (Cover & Thomas, 2006). Entropy has also been applied to characterize market structure, where higher entropy values often indicate greater disorganization and lower predictability (Olbrys, 2022). Empirical studies show that entropy measures are sensitive to regime shifts and can detect transitions between stable and turbulent market conditions. For instance, entropy has been used to assess stock market efficiency during crises, capturing significant increases in uncertainty during systemic stress (Papla & Siedlecki, 2024), and complexity–entropy frameworks have been applied to cryptocurrency markets, revealing nonlinear transitions and emergent order during crash dynamics (Bui et al., 2025). Dionisio et al. (2007) quantified uncertainty and disorder in financial markets using entropy as an alternative to traditional risk measures, while Topal (2021) applied an entropy-based multi-criteria approach to evaluate electricity generation companies under uncertainty. Riabikh et al. (2025) introduced an entropy-based text feature engineering method for forecasting financial liquidity, demonstrating the value of information-theoretic measures in predictive financial analytics. Collectively, these studies highlight entropy’s ability to capture both randomness and structural evolution in financial systems.

Advances in financial thermodynamics and market microstructure analysis have provided physically motivated definitions of entropy. Recent studies show that models incorporating thermodynamic variables from limit order book dynamics can use entropy to measure market disorder, trading activity, and liquidity (Li et al., 2024). Stochastic thermodynamics further conceptualizes financial markets as non-equilibrium systems with entropy production and dissipation, analogous to physical systems governed by the Second Law of Thermodynamics. Touzo et al. (2021) applied information thermodynamics to the Glosten–Milgrom model, quantifying information flows in financial markets, while Ghosh (2025) reviewed statistical mechanics in financial thermodynamic systems, illustrating how entropy and energy analogies can model market dynamics and systemic risk.

Despite these developments, mainstream financial modeling remains dominated by econometric approaches, particularly ARCH and GARCH models, which effectively capture volatility clustering and conditional heteroskedasticity in time series (Engle, 1982; Bollerslev, 1986). However, these models assume linear or weakly nonlinear stochastic processes and lack a physical interpretation of irreversibility, dissipation, or information flow. Francq et al. (2011) examined variance-targeting techniques in GARCH models, highlighting their advantages and limitations, while Kim and Won (2018) proposed a hybrid approach combining LSTM neural networks with multiple GARCH models to improve volatility forecasting. Currently, there is limited research integrating information diffusion, nonlinear interactions, and time-dependent external disturbances into a unified framework, despite clear analogies between financial markets and physical transport systems where information diffuses, interacts nonlinearly, and is subject to external perturbations.

The present study develops a thermofluid-inspired nonlinear entropy model for financial markets, formulating financial entropy evolution as a nonlinear reaction–diffusion equation. This framework captures how information spreads, agents interact, and external shocks influence market dynamics. Drawing analogies from heat and mass transfer, the model incorporates nonlinear reaction terms to represent feedback, amplification, and saturation effects in market behavior, alongside a time-dependent source term for external disturbances. The originality of the study lies in combining thermodynamic principles, nonlinear dynamics, and financial modeling within a single partial differential equation framework. The model demonstrates how diffusion, nonlinearity, and external forces interact, offering deeper insights into market inefficiency, risk propagation, and systemic instability than existing entropy-based descriptive analyses. This work advances thermodynamic financial modeling and establishes a robust framework for studying complex financial systems under nonequilibrium conditions.

2. Thermodynamic Foundation and Financial Systems Analogy

The current study is based on the principles of the Second Law of Thermodynamics, which deals with the evolution of entropy in non-reversible systems. The balance equation of entropy in continuum thermodynamics can be written as

$$\frac{\partial s}{\partial t} + \nabla \cdot \mathbf{J}_s = \dot{s}_{gen}, \quad (1)$$

where s denotes entropy per unit volume, $\mathbf{J}_s$ is the entropy flux, and $\dot{s}_{gen}$ represents the local entropy generation due to irreversible processes.

In classical heat transfer and fluid flow, entropy generation arises primarily from thermal gradients and viscous dissipation. The local volumetric entropy generation rate is typically given by

$$\dot{s}_{gen} = \frac{k}{T^2} |\nabla T|^2 + \frac{\mu}{T} \Phi_v, \quad (2)$$

where k is the thermal conductivity, T is the absolute temperature, μ is the dynamic viscosity, and Φ_v denotes the viscous dissipation function. In equation (2), the first term in describes entropy generation due to heat transfer across finite temperature gradients, and the second term accounts for irreversibility associated with viscous effects in fluid motion.

Table 1. Relations between thermodynamic and financial variables

Thermodynamic System	Financial System
Temperature (T)	Financial entropy (S)
Thermal gradient (∇T)	Information gradient (∇S)
Heat flux	Information/market flow
Thermal diffusivity (α)	Financial diffusivity (β)
Viscous dissipation	Market friction and regulation ($P(t)$)
External heating	Market shocks ($\Phi(y, t)$)

On the basis of these thermodynamic laws, the financial market is considered as an irreversible system where uncertainty develops through diffusion, nonlinear interactions, and external forcing. In this context, financial entropy $S(y, t)$ describes the level of disorder or uncertainty in the financial system.

The correspondence between thermodynamic and financial variables is established as follows:

Under this analogy, entropy generation due to thermal gradients, represented by $(\nabla T)^2$, is extended to financial systems as a gradient-driven entropy production term $(\nabla S)^2$. This term captures the effect of spatial heterogeneity and information asymmetry within the market.

2.1 Entropy-Based Model Justification

By invoking a Fick-type constitutive relation for entropy flux,

$$\mathbf{J}_S = -\beta \nabla S, \quad (3)$$

the divergence of the entropy flux yields a diffusion term:

$$\nabla \cdot \mathbf{J}_S = \beta \nabla^2 S. \quad (4)$$

Furthermore, inspired by classical entropy generation formulations, the total entropy production in the financial system is modeled as a combination of gradient-driven and nonlinear contributions:

$$\dot{S}_{gen} = \beta |\nabla S|^2 + aS - bS^2. \quad (5)$$

Consequently, the governing equation for financial entropy evolution is derived as

$$\frac{\partial S}{\partial t} = \beta \nabla^2 S + \beta |\nabla S|^2 + aS - bS^2 + \Phi(y, t) - P(t), \quad (6)$$

which is consistent with entropy transport equations in thermofluid systems, extended to incorporate financial dynamics. The resulting model captures key mechanisms analogous to thermodynamic systems:

- Diffusion term $\beta \nabla^2 S$ represents the spread of information across the market.
 - Gradient term $\beta |\nabla S|^2$ accounts for entropy generation due to market heterogeneity and information imbalance.
 - Nonlinear term $aS - bS^2$ models endogenous market interactions such as speculation and saturation.
 - Source term $\Phi(y, t)$ represents external shocks such as economic news or policy changes.
 - Dissipation term $P(t)$ captures regulatory interventions and transaction costs that stabilize the system.
- Thus, this model establishes a thermodynamically consistent framework for financial modeling, in which market dynamics are interpreted as an irreversible entropy-driven process.

2.2 Model Assumptions

The formulation of the thermofluid-inspired financial entropy model is based on the following assumptions:

- The financial market is idealized as a one-dimensional domain ($y \in [0, L]$), representing market depth or the hierarchy of information dissemination. This simplification captures dominant directional information flow while maintaining analytical tractability, as commonly adopted in econophysical modeling.
- Financial entropy $S(y, t)$ is assumed to be a continuous, bounded, and differentiable scalar field, enabling the use of continuum mechanics tools to describe aggregate market uncertainty.
- Information diffusion is modelled as a Fickian process with constant financial diffusivity (β), reflecting homogeneous average market conditions and enabling isolation of nonlinear effects.
- Entropy generation arises from spatial gradients and internal interactions, represented through diffusion and nonlinear reaction terms, consistent with nonequilibrium thermodynamic systems.

- The nonlinear reaction term $(aS - bS^2)$ follows a logistic structure, capturing the balance between speculative amplification ((a)) and market saturation or stabilizing constraints ((b)).
- External disturbances are incorporated via a time- and space-dependent source term $\Phi(y, t)$, representing heterogeneous and dynamic financial shocks.
- The dissipation term $P(t)$ models regulatory interventions and systemic stabilizing mechanisms acting uniformly across the market.
- The system is assumed isothermal and closed, neglecting stochastic fluctuations and memory effects to focus on deterministic entropy evolution at the macroscopic scale.
- Boundary conditions correspond to fixed entropy levels or no-flux constraints, reflecting varying degrees of market openness and information exchange.

3. Mathematical Model

Based on an entropy balance principle, the evolution of financial entropy is governed by

$$\frac{\partial S}{\partial t} = \beta \frac{\partial^2 S}{\partial y^2} + \beta \left(\frac{\partial S}{\partial y} \right)^2 + aS - bS^2 + \Phi(y, t) - P(t), \quad (7)$$

where:

- $S(y, t)$: financial entropy (measure of market uncertainty),
- y : spatial coordinate representing market depth,
- t : time,
- β : financial diffusivity (rate of information spread),
- $\beta \frac{\partial^2 S}{\partial y^2}$: entropy diffusion term,
- $\beta \left(\frac{\partial S}{\partial y} \right)^2$: entropy generation due to spatial gradients (market heterogeneity),
- a : linear entropy growth coefficient (speculative amplification),
- b : nonlinear saturation coefficient (market stabilization),
- $\Phi(y, t)$: external entropy source (market shocks),
- $P(t)$: entropy dissipation function (regulatory and frictional effects).

The external shock model for the present study is modeled as

$$\Phi(y, t) = Ae^{-kt}(1 + \delta y) \quad (8)$$

Where A : shock intensity, k describes the temporal decay rate of shocks, and δ : is the spatial heterogeneity parameter across market depth.

To generalize the governing equation, the following dimensionless variables are introduced:

$$S^* = \frac{S}{S_0}, y^* = \frac{y}{L}, t^* = \frac{t}{t_0}, \quad (9)$$

where: S_0 is the characteristic entropy scale, L connotes characteristic market depth, and t_0 describes the characteristic time scale.

3.1 Dimensionless Governing Equation

We introduced dimensionless variables (9) into the governing equation to obtain:

$$\frac{\partial S^*}{\partial t^*} = Fo \frac{\partial^2 S^*}{\partial y^{*2}} + Fo \left(\frac{\partial S^*}{\partial y^*} \right)^2 + \lambda S^* - \gamma S^{*2} + \Phi^*(y^*, t^*) - P^*(t^*), \quad (10)$$

where the dimensionless parameters are defined as

$$Fo = \frac{\beta t_0}{L^2}, \lambda = at_0, \gamma = bS_0 t_0. \quad (11)$$

The dimensionless source and dissipation terms are given by

$$\Phi^*(y^*, t^*) = \frac{t_0}{S_0} \Phi(y, t), P^*(t^*) = \frac{t_0}{S_0} P(t). \quad (12)$$

Accordingly, the dimensionless source term becomes

$$\Phi^*(y^*, t^*) = \Pi e^{-Kt^*} (1 + \Delta y^*), \quad (13)$$

where

$$\Pi = \frac{At_0}{S_0}, K = kt_0, \Delta = \delta L. \quad (14)$$

The initial condition is prescribed as

$$S^*(y^*, 0) = S_i(y^*), \quad (15)$$

where $S_i(y^*)$ represents a bounded initial entropy distribution. The boundary conditions are specified as

$$S^*(0, t^*) = 0, S^*(1, t^*) = 1, \quad (16)$$

representing fixed entropy states at the market boundaries. Alternatively, a no-flux boundary condition may be imposed:

$$\left. \frac{\partial S^*}{\partial y^*} \right|_{y^*=0,1} = 0, \quad (17)$$

which corresponds to no information exchange across the market boundaries.

3.2 Numerical Simulation

The dimensionless governing equation is solved numerically using an explicit finite difference method. The spatial and temporal domains are discretized as:

$$y_i^* = i\Delta y, t_n^* = n\Delta t, \quad (18)$$

where Δy and Δt are the spatial and temporal step sizes, respectively. The derivatives are approximated as:

$$\left\{ \begin{array}{l} \frac{\partial S^*}{\partial t^*} \approx \frac{S_i^{n+1} - S_i^n}{\Delta t}, \\ \frac{\partial^2 S^*}{\partial y^{*2}} \approx \frac{S_{i+1}^n - 2S_i^n + S_{i-1}^n}{(\Delta y)^2}, \\ \frac{\partial^2 S^*}{\partial y^{*2}} \approx \frac{S_{i+1}^n - 2S_i^n + S_{i-1}^n}{(\Delta y)^2}, \\ \left(\frac{\partial S^*}{\partial y^*} \right)^2 \approx \left(\frac{S_{i+1}^n - S_{i-1}^n}{2\Delta y} \right)^2. \end{array} \right. \quad (19)$$

The resulting finite difference scheme is given by

$$\begin{aligned} S_i^{n+1} = & S_i^n + Fo \frac{\Delta t}{(\Delta y)^2} (S_{i+1}^n - 2S_i^n + S_{i-1}^n) + Fo \Delta t \left(\frac{S_{i+1}^n - S_{i-1}^n}{2\Delta y} \right)^2 \\ & + Fo \Delta t \left(\frac{S_{i+1}^n - S_{i-1}^n}{2\Delta y} \right)^2 + \Delta t (\lambda S_i^n - \gamma (S_i^n)^2) + \Delta t \Phi_i^n - \Delta t P^n. \end{aligned} \quad (20)$$

where

$$\Phi_i^n = \Pi e^{-Kt_n}(1 + \Delta y_i), P^n = P(t_n).$$

For numerical stability, the following conditions must be satisfied:

$$\begin{cases} Fo \frac{\Delta t}{(\Delta y)^2} \leq \frac{1}{2}, \\ \Delta t \cdot \max \left| \frac{\partial S^*}{\partial y^*} \right|^2 \ll 1. \end{cases} \quad (21)$$

The computational algorithm takes the following steps

- Initialize $S_i^0 = S_i(y^*)$ for all spatial grid points.
- Apply boundary conditions at each time step.
- Compute diffusion, entropy generation, reaction, source, and dissipation terms.
- Update the solution using the explicit scheme.
- Advance the time level: $n \rightarrow n + 1$.
- Repeat until the final simulation time is reached.

The present formulation extends conventional financial diffusion models by incorporating explicit entropy generation mechanisms, thereby providing a thermodynamically consistent framework for analyzing irreversible dynamics and shock propagation in financial markets.

Table 2. Dimensionless parameters and their interpretations

Symbol	Description	Typical Values / Range	Dimensionless Form
$S(y, t)$	Financial entropy	0 – 1	$S^* = S/S_0$
β	Financial diffusivity	0.01 – 0.5	$Fo = \beta t_0/L^2$
a	Linear growth coefficient	0.5 – 2	$\lambda = at_0$
b	Nonlinear saturation	0.1 – 1	$\gamma = bS_0t_0$
A	Shock intensity	0.5 – 5	$\Pi = At_0/S_0$
k	Shock decay rate	0.1 – 1	$K = kt_0$
δ	Spatial variation	0 – 1	$\Delta = \delta L$
$P(t)$	Dissipation / stabilization	0.05 – 0.2	$P^*(t^*)$

4. Results and Discussion

To further understanding the behaviour of the embedded parameters on the financial entropy distribution, some graphs have been plotted in this section with appropriate discussion of the results.

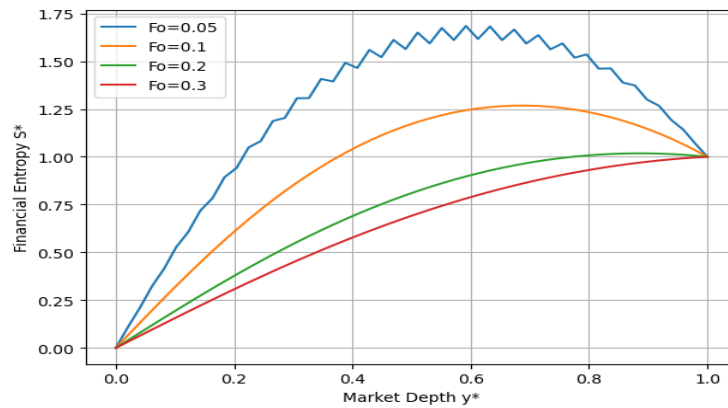


Fig. 1. Financial entropy versus market depth for various values of diffusion parameter F_0

Fig. 1 presents the spatial variation of financial entropy with market depth for different values of the diffusion parameter (Fo). The results show that lower diffusion ($Fo = 0.05$) leads to a pronounced entropy peak (≈ 1.7) around ($y^* \approx 0.5 - 0.6$), indicating localized accumulation of uncertainty within the market. This suggests that weak information diffusion allows entropy to concentrate in specific regions. As (Fo) increases ($0.2 - 0.3$), the entropy profiles become smoother, and the peak magnitude reduces significantly (≈ 1.0). This reflects enhanced information spreading, which redistributes uncertainty more uniformly and suppresses localized instability. The observed behaviour highlights a diffusion–reaction balance: diffusion governs the spatial redistribution of entropy, while nonlinear reactions control its growth and saturation. Interior peaks arise in regions where entropy generation temporarily exceeds dissipation; however, these remain bounded due to nonlinear effects. Overall, higher diffusivity promotes stability by preventing the formation of concentrated entropy zones.

The implication is that higher financial diffusivity enhances market stability by mitigating localized risk concentrations, suggesting that improved information flow, transparency, and liquidity can reduce the likelihood of regionalized market disruptions.

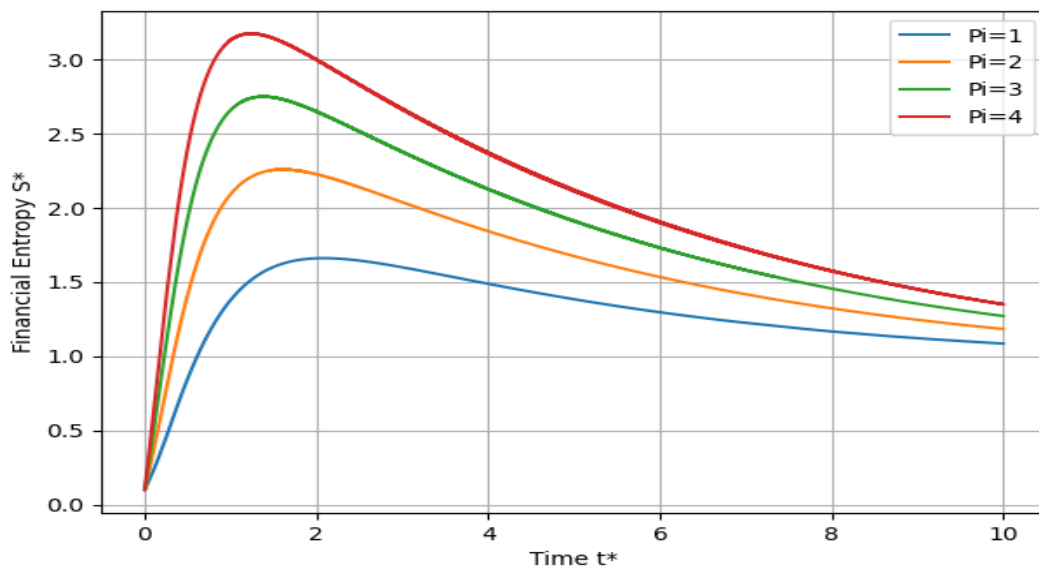


Fig. 2. Effects of shock intensities Π on financial entropy S^*

Fig. 2 illustrates the temporal evolution of financial entropy (S^*) at a fixed market depth for varying shock intensities (Π). The results show that, at early times, entropy growth is strongly driven by external shocks, with higher (Π) producing a more rapid increase in uncertainty. This reflects the immediate impact of exogenous disturbances on market instability. As time progresses, nonlinear saturation and dissipation mechanisms become dominant, limiting further entropy growth. Consequently, peak entropy increases with shock intensity, but at a diminishing (sublinear) rate, indicating the presence of stabilizing internal dynamics. At longer times, all profiles converge to a common steady state, independent of (Π). This confirms that external shocks have only transient effects, while the long-term market behavior is governed primarily by intrinsic reaction–dissipation processes. The exponential decay of the shock term further reinforces this transition from a shock-dominated regime to a stable equilibrium state.

Fig. 3 presents the temporal evolution of financial entropy (S^*) at different market depths ($y^* = 0.24, ; 0.51, ; 0.76$). The results reflect the combined influence of diffusion, nonlinear reaction, and time-decaying external shocks. This figure clearly demonstrates the interplay between space and time: The spatial effect y^* controls the magnitude of entropy generation via $(1 + \Delta y_i)$ while the temporal effect (t^*) controls decay of shocks. Combined effect shows that at early stage, the spatial differences amplified, whereas and at later stage, the temporal decay dominates, reducing differences.

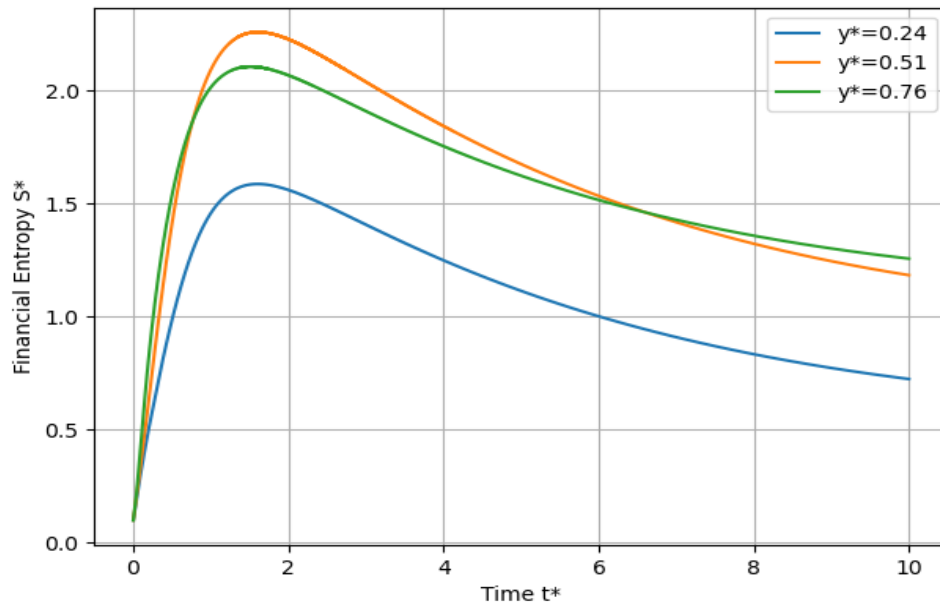


Fig. 3. Distribution of temporary evolution of financial entropy (S^*) at different market depths y^*

The time evolution of financial entropy demonstrates significant spatial dependence, especially during the shock-dominated regime. The analytical results indicate that the rate of entropy production increases with the level of market depth owing to the spatially varying source term, causing a faster rate and higher peaks in the entropy evolution for the interior regions. The maximum entropy is observed when the nonlinear saturation and dissipation are balanced by the level of external forcing, and the region that demonstrates the maximum peak is the mid-domain region. As the shock term decays exponentially, a reaction-dissipation regime is established, causing a decrease in entropy to a steady state.

Spatio-temporal Evolution of Financial Entropy

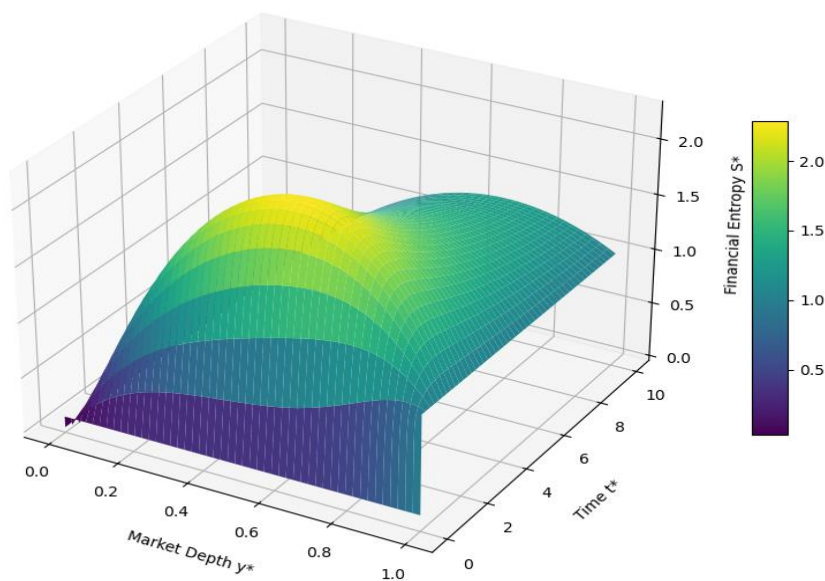


Fig. 4. Spatio-Temporal Evolution of Financial Entropy

Fig. 4 illustrates the joint variation of financial entropy (S^*) with market depth (y^*) and time (t^*). The results reveal a nonlinear and non-monotonic dependence on depth, with entropy attaining a maximum at intermediate liquidity levels, forming a distinct ridge. As time progresses, the overall entropy magnitude decreases and the surface becomes smoother, indicating progressive stabilization.

The coupled dependence on (y^*) and (t^*) is inherently nonlinear and inseparable. At early times, entropy is highly sensitive to variations in market depth, reflecting strong liquidity-driven heterogeneity. This sensitivity diminishes over time as diffusion and nonlinear saturation effects dominate, reducing spatial disparities. The evolution of the entropy ridge suggests a self-organizing mechanism in which the system adjusts toward a moderate and stable entropy state.

From an economic perspective, the peak at intermediate liquidity aligns with the notion that partial liquidity maximizes informational frictions and order-flow complexity, while very low or high liquidity reduces uncertainty. The temporal decline in entropy indicates adaptive market behaviour, such as learning, arbitrage, and regulatory effects driving the system toward equilibrium. This suggests that policies enhancing market efficiency can accelerate stabilization and reduce persistent uncertainty.

Overall, the findings demonstrate that financial entropy is jointly governed by liquidity structure and temporal dynamics, and that time-dependent entropy surfaces offer a more comprehensive and realistic framework for capturing market complexity than static measures, with important implications for modeling, forecasting, and risk assessment.

5. Conclusion

This study proposes a thermofluid-based nonlinear entropy approach to study the behaviour of financial uncertainty under diffusion, nonlinear interactions, and external time-dependent shocks. The findings show that financial entropy evolves through a complex interplay of spatial diffusion, nonlinear saturation, and external temporal shocks. The main findings are listed below:

- The financial entropy is controlled by a diffusion-reaction-dissipation balance, and this determines its spatial and time evolution.
- The rate of diffusion is inversely proportional to entropy accumulation, and higher diffusion results in a stable distribution.
- The effect of shocks is significant for entropy levels, but this effect decreases over time because of exponential decay.
- The entropy levels are maximum for a particular range of the market, and this is the region of high uncertainty.
- The system is nonlinear and saturated, and this ensures that the entropy remains finite and never goes to infinity.
- The time evolution of entropy is such that there is a buildup, a maximum, and then a decay to equilibrium, and this is consistent with adaptive evolution.
- The entropy surface shows that the space and time are coupled and that this is a non-separable surface, and this is consistent with the behaviour of the markets.

5.1 Applications and Implications

- Provides a quantitative framework for measuring financial instability and systemic risk using entropy-based metrics.
- Useful for identifying critical liquidity regimes where uncertainty is maximized.
- Offers insights for regulators and policymakers to design interventions that enhance market stability through improved information diffusion.
- Applicable to risk management and portfolio analysis, where entropy can complement traditional volatility measures.
- Establishes a foundation for physics-informed financial modeling, linking thermodynamic principles with economic dynamics.

6. Limitations

The current model is based on a one-dimensional market depth, while markets are generally n-dimensional. The parameters are deterministic, while there are stochastic variations in the markets. More so, the external shock is assumed to be an exponentially decaying function, while this may not be the case in real markets. The empirical validation is also not included in this study.

7. Future Work

The present model can be extended to n-dimensional markets, including network effects and inter-market interactions with inclusion of Stochastic terms. Also, the model can be validated and calibrated using real financial datasets with shock function assumed to be a regime-switching or non-exponentially decaying function, and the model can also be extended to include machine learning concepts.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

Competing Interests

Authors have declared that no competing interests exist.

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